

# STATISTICAL DOWNSCALING OF THE SIGNIFICANT WAVE HEIGHT

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# OUTLINE

- ❑ Motivation
- ❑ Objectives and challenges
- ❑ Existing work
- ❑ The model
- ❑ Results
- ❑ Conclusion and perspectives

# MOTIVATION

- ❑ Wave climate characterization is important for a wide number of marine activities
- ❑ GCMs (general circulation models) provide future projections for atmospheric variables with coarse spatial resolution
- ❑ GCMs simulate wind but not sea state parameters
- ❑ Statistical and dynamical downscaling models bridge the gap between GCM simulations and decision makers requirements

- ❑ Statistical downscaling models construct an empirical relationship between large scale and local scale variables using historical data
- ❑ Assuming that this relationship is stationary, future projections can be made using GCM simulations of the large scale variables
- ❑ This makes statistical downscaling models computationally efficient
- ❑ For a rigorous comparison between statistical and dynamical methods we refer to the studies Wang et al. (2010)[1] and Laugel et al. (2014)[2]



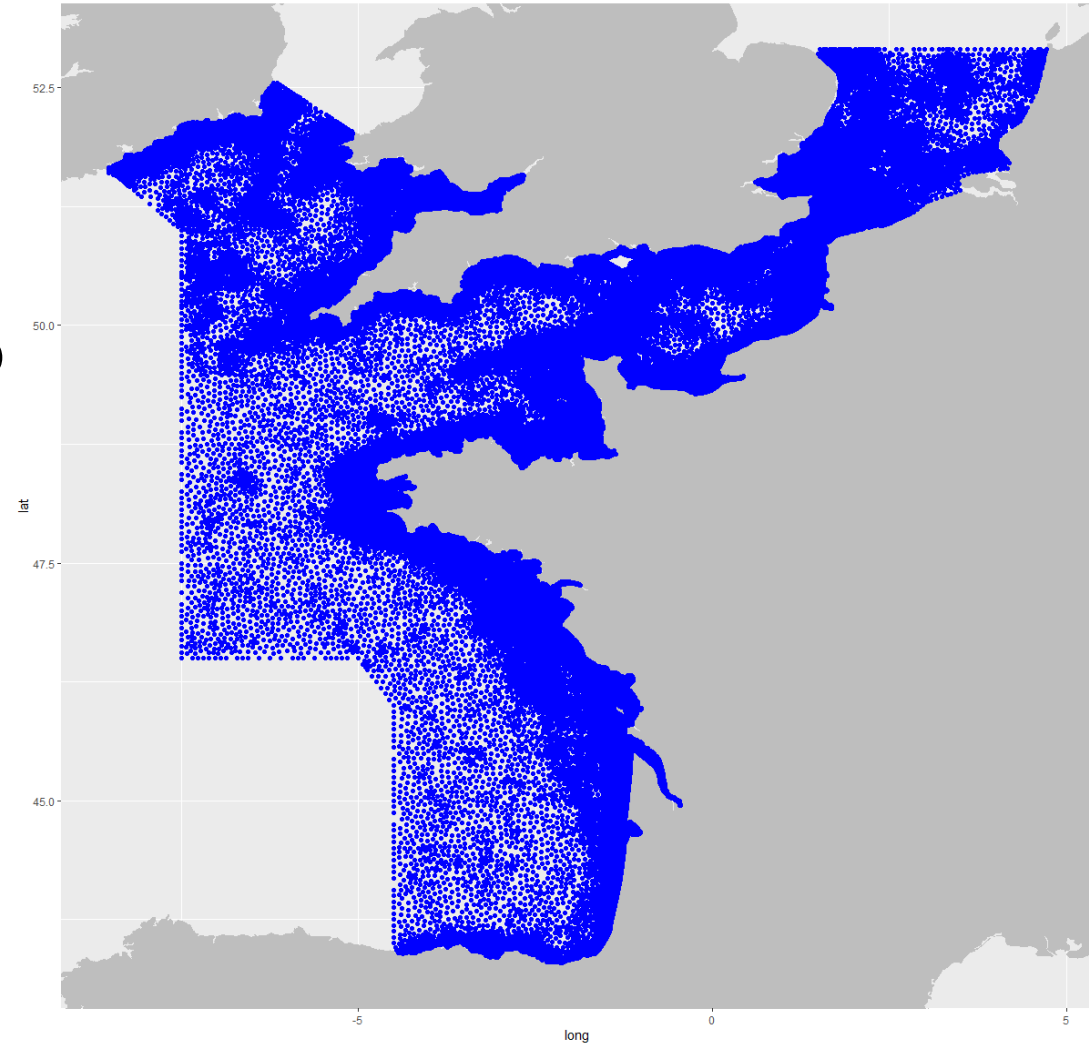
Before using climate model simulations for making projections with statistical downscaling models, bias correction methods are needed. And this is not the focus of this study

# OBJECTIVES

- ❑ Establish a statistical link function between the wind and the significant wave height
  - ❑ **Predictand:** the significant wave height ( $H_s$ ) from the hindcast database Homere
  - ❑ **Predictors:** wind data from the ERA5 reanalysis dataset
  - ❑ **Method:** linear regression with a suitable penalization method
- ❑ The relationship has to be physically interpretable

# DATA

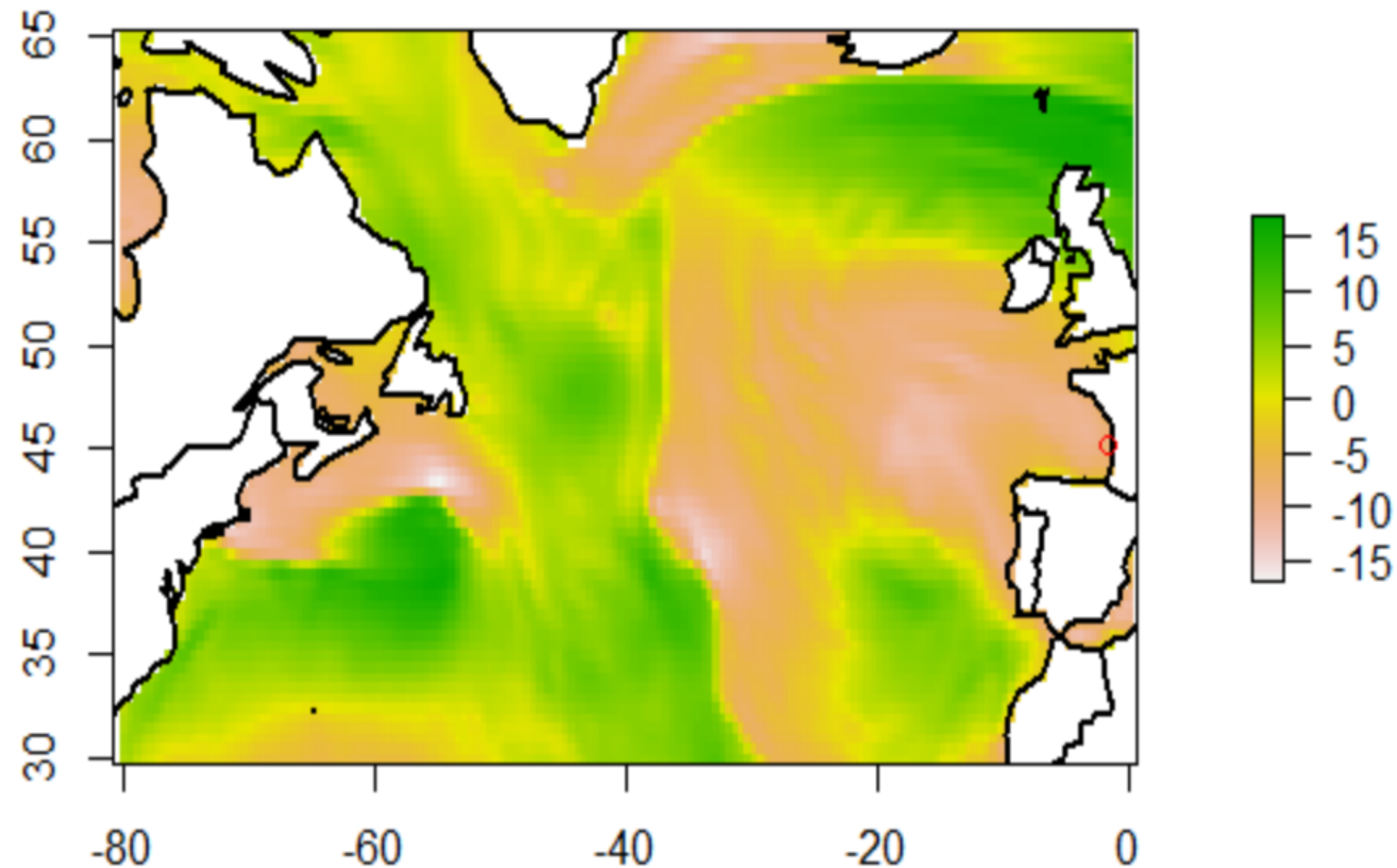
- ❑ Hindcast database Homere:
  - ❖ Sea state hindcast database, based on WAVEWATCH III
  - ❖ High resolution grid with one hour time step
  - ❖ From 1994 to 2019
  - ❖ The wind forcing: CFSR
  
- ❑ ERA5 reanalysis database:
  - ❖ Hourly wind components with  $0.25^{\circ} \times 0.25^{\circ}$  spatial resolution



# CHALLENGES

- ❑ High dimensionality of the input space
- ❑ Multicollinearity
- ❑ The statistical downscaling model has to take into consideration the sea state composition (wind sea – swells)
- ❑ Non-instantaneous and non-local relationship between wind and waves

The zonal wind component



The location of interest is situated at the Bay of Biscay at 45.2°N, 1.6°W

# EXISTING WORK

- ❑ Camus et al. (2014b)[3] used a weather types model to downscale wave parameters in north-west of Spain. To account for the swell composition, the predictor was defined as the three-daily mean of sea level pressure and pressure gradients
- ❑ Perez et al. 2014[4] proposed a method, called ESTELA, that defines the wave generation area and wave travel time at any location worldwide
- ❑ Camus et al.(2014a)[5] and Herermiller et al. (2016)[6] used the ESTELA approach to define the predictors for their statistical downscaling approach



# THE MODEL

$$H_s = X_L \hat{\beta}_L + X_G \hat{\beta}_G + \epsilon \quad (1)$$

- $H_s$ : significant wave height
- $X_L$  et  $X_G$ : local and global predictors
- $\hat{\beta}_L$  et  $\hat{\beta}_G$ : local and global coefficients
- $\epsilon$ : model error

# THE PREDICTORS

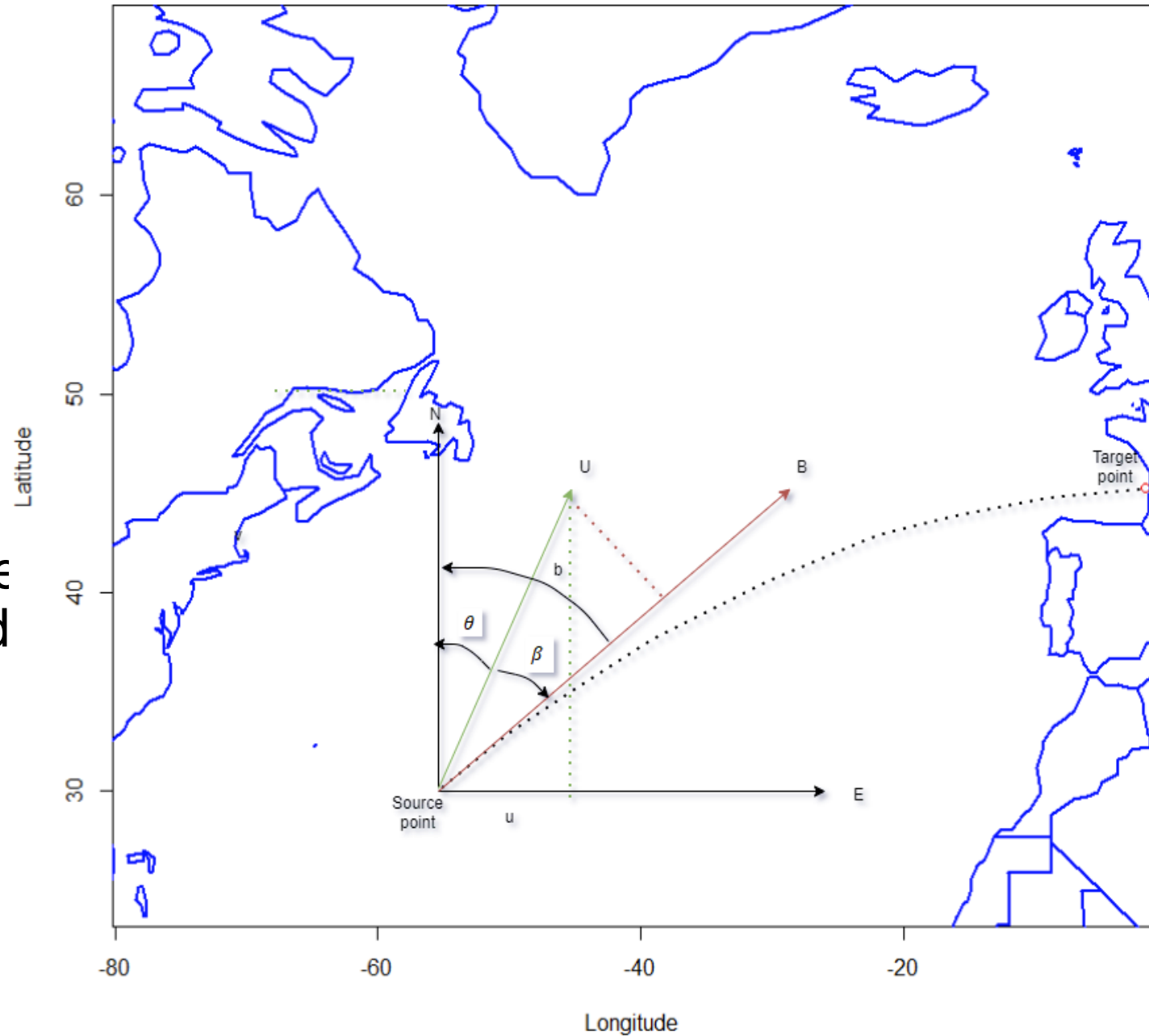
- **The local predictor  $x^L$**  is defined based on the wind speed and the fetch at the local point
- **The global predictor  $x^G$**  is defined as the projected wind: the wind components at each grid point are projected into the bearing of the target point in a great circle path

$$W = \sqrt{u^2 + v^2} \cos^2(1/2 \beta)$$

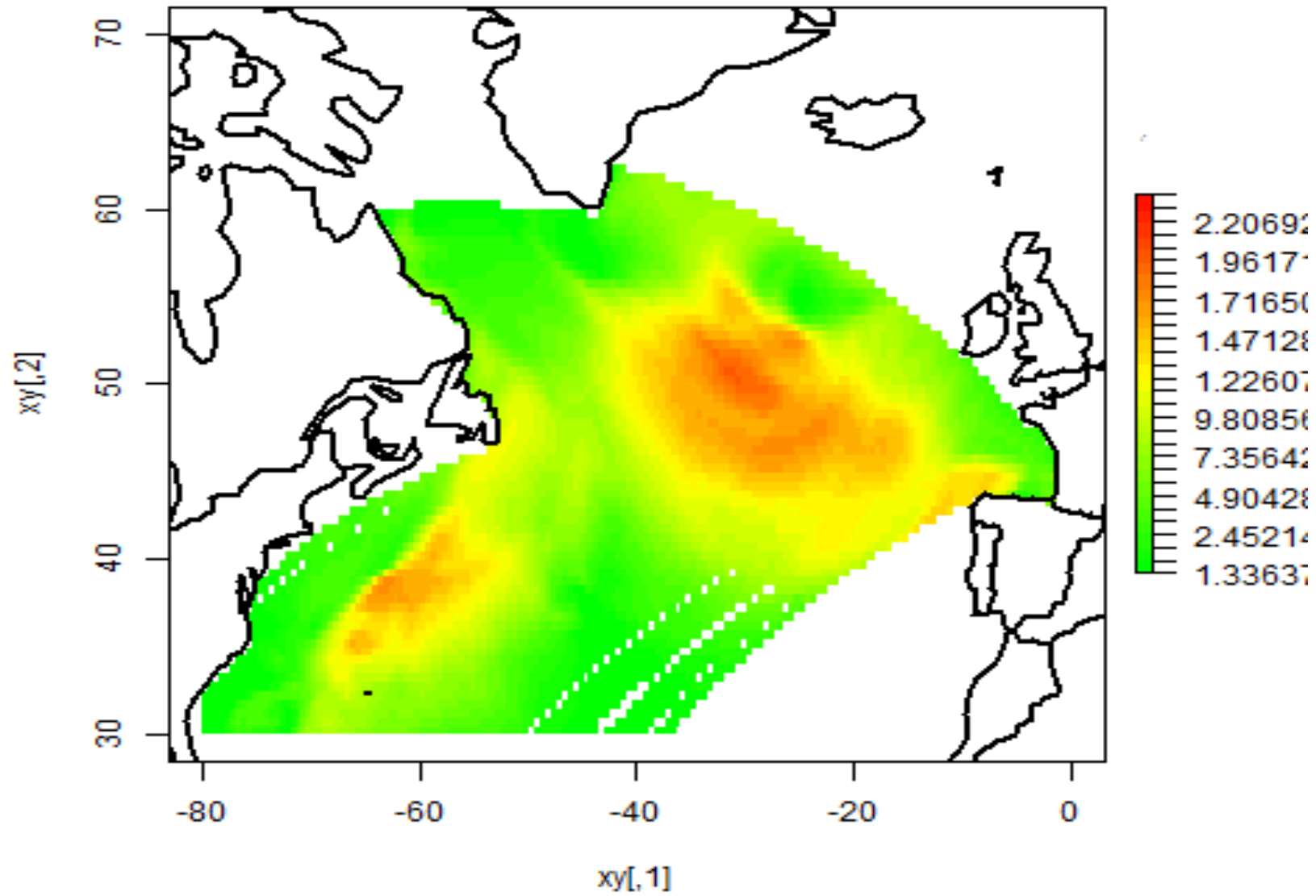
$$\beta = b - \theta$$

$$\theta = \text{atan}(u/v)$$

□ where  $W$  is the projected wind,  $u$  and  $v$  the wind components,  $b$  is the great circle bearing, and  $\theta$  is the wind direction



# The projected wind



- **The spatial coverage of  $X \uparrow G$ :** assuming that waves travel along a great circle path, grid points whose path is blocked by land are neglected
- **The temporal coverage of  $X \uparrow G$**  is defined by two parameters, called travel time of waves  $t \downarrow j$  and the temporal width  $\alpha \downarrow j$ , using a fully data-driven approach
- At time  $t$  the global predictor  $X \uparrow G$  is defined as:

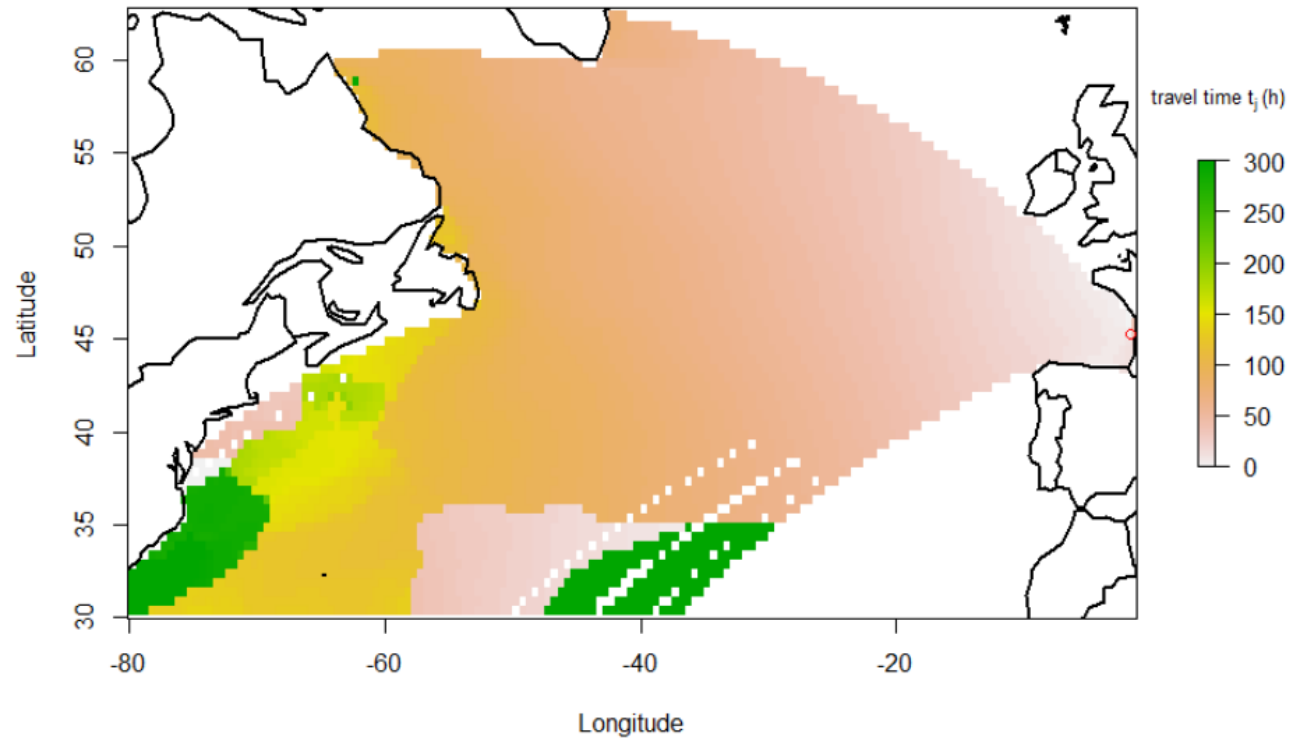
$$X \uparrow G (t) = \{ \bar{W} \downarrow 1 \uparrow 2 (t - t \downarrow 1 - \alpha \downarrow 1 : t - t \downarrow 1 + \alpha \downarrow 1), \dots, \bar{W} \downarrow j \uparrow 2 (t - t \downarrow j - \alpha \downarrow j : t - t \downarrow j + \alpha \downarrow j), \dots, \bar{W} \downarrow m \uparrow 2 (t - t \downarrow m - \alpha \downarrow m : t - t \downarrow m + \alpha \downarrow m) \}$$

- Where  $\bar{W} \downarrow j \uparrow 2 (t - t \downarrow j - \alpha \downarrow j : t - t \downarrow j + \alpha \downarrow j)$  is the mean of the squared projected wind at location  $j$  in a time window controlled by  $\alpha \downarrow j$  and  $t \downarrow j$

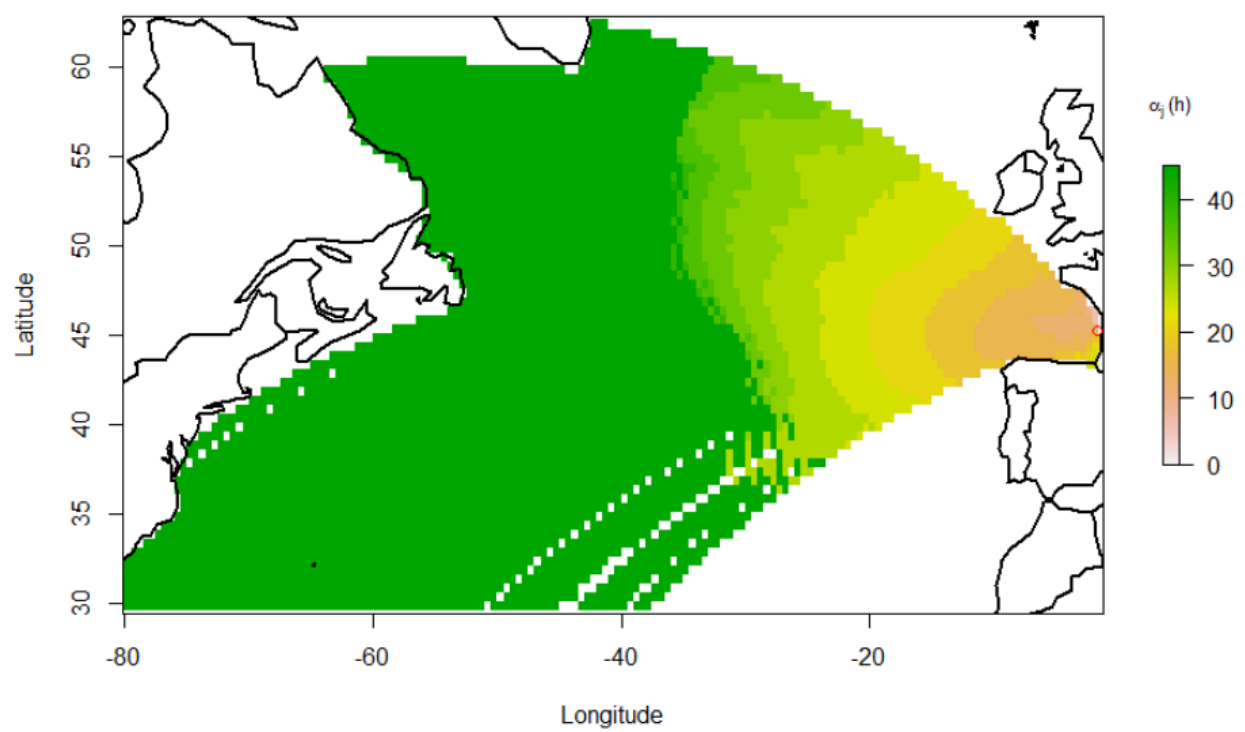
□  $\alpha_{lj}$  and  $t_{lj}$  are estimated as follows :

$$(t_{lj}, \alpha_{lj}) = \underset{t, \alpha}{\operatorname{argmax}} \operatorname{cor}(H_s, \bar{W}_{lj} \uparrow 2 (t - t_{lj} - \alpha_{lj} : t - t_{lj} + \alpha_{lj}))$$

The estimated travel time  $t_{ij}$



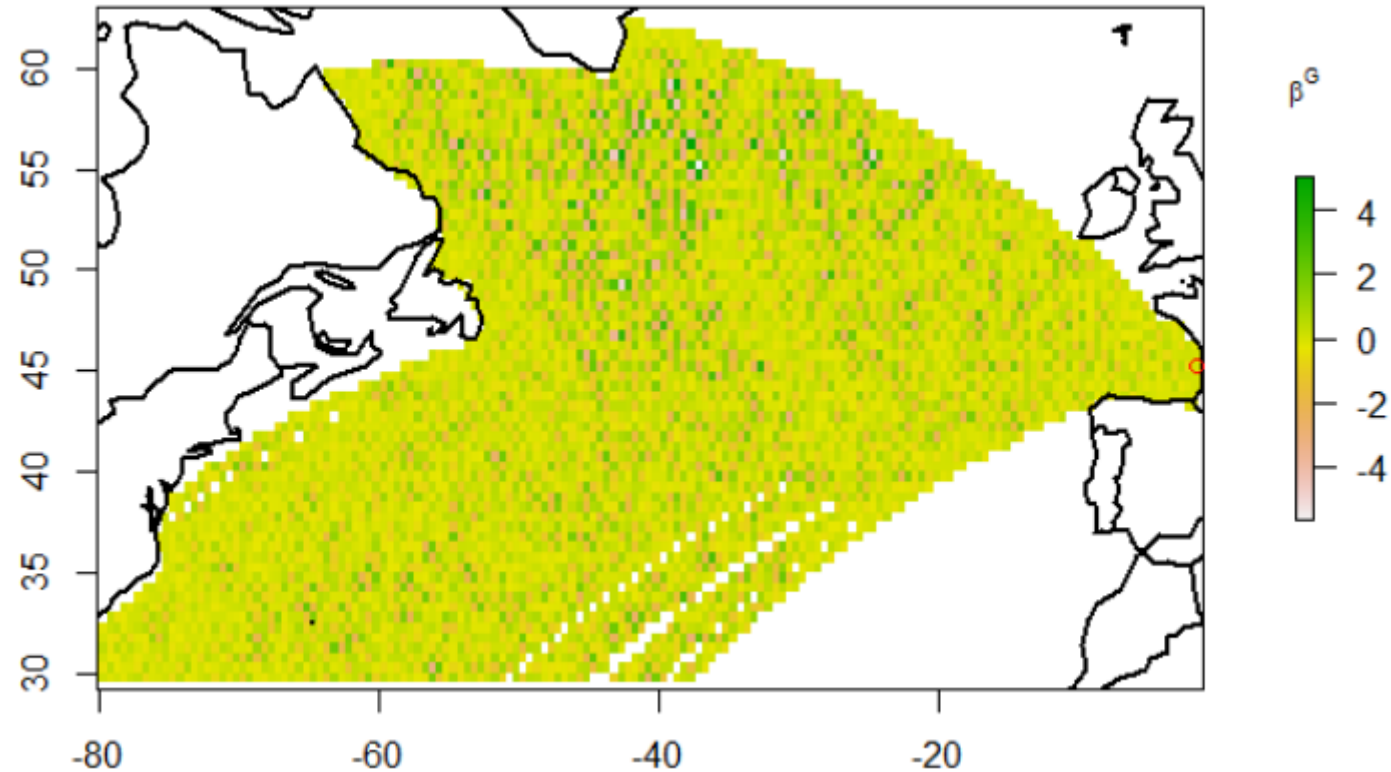
The estimated temporal width  $\alpha_{ij}$



# MODEL ESTIMATION

□ The model  $Y = X\beta + \epsilon$  can be estimated using least squares so that  $\hat{\beta}_{ls} = (X^T X)^{-1} X^T Y$

- However, in the case of high multicollinearity, the matrix  $X^T X$  may be ill-conditioned
- The least squares estimates have low bias and high variance which affects the prediction accuracy of the model





□ To address this issue, Ridge regression shrinks the coefficients by imposing a penalty on the residual sum of squares so that :

$$\hat{\beta}_{ridge} = \operatorname{argmin} \|X\beta - Y\|_2^2 + \lambda \|\beta\|_2^2 \quad (2)$$

□ The solution of (2) is  $\hat{\beta}_{ridge} = (X^T X + \lambda I)^{-1} X^T Y$ . Ridge thus, adds positive elements to the diagonal of  $X^T X$  before inversion

□ In fact Ridge shrinks all the EOFs of  $X$  and a high amount of shrinkage is applied to EOFs with small variance

□ We extend this to the general case where  $\hat{\beta}_{Eridge} = (X^T X + \lambda \Delta)^{-1} X^T Y$  where  $\Delta$  is the penalty matrix.  $\Delta$  can be interpreted as a prior on  $\beta$

□ LASSO is another shrinkage method. Instead of using the norm 2 in (2) it uses the norm 1 so that

$$\hat{\beta}_{lasso} = \operatorname{argmin} \|X\beta - Y\|_2^2 + \lambda \|\beta\|_1$$

# MODEL ESTIMATION

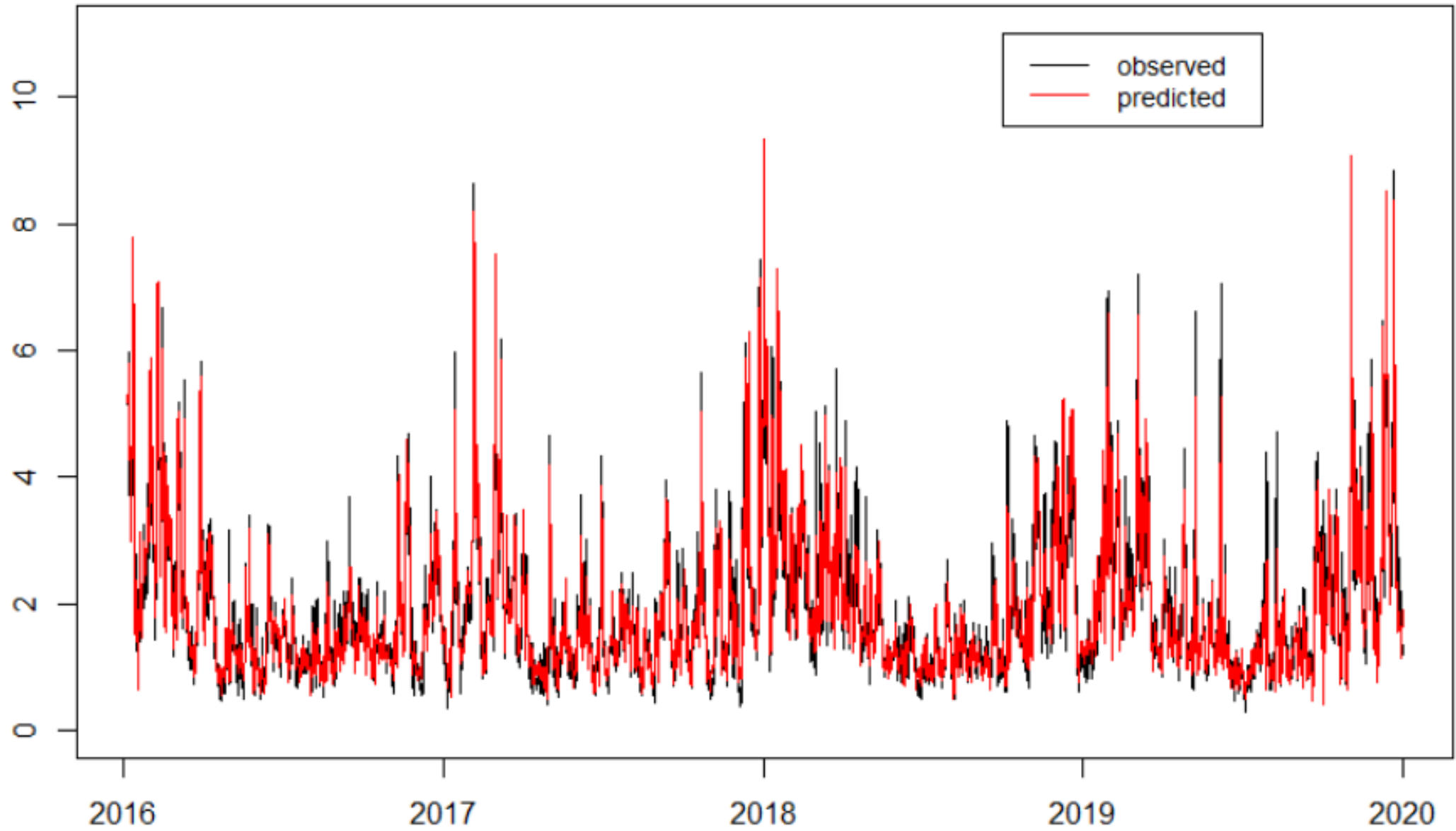
□ The penalized version of the model (1) can be written as:

$$(\hat{\beta}^L, \hat{\beta}^G) = \operatorname{argmin} \|X^L \hat{\beta}^L + X^G \hat{\beta}^G - Hs\|^2 + \lambda \hat{\beta}^G{}^T \Delta \hat{\beta}^G$$

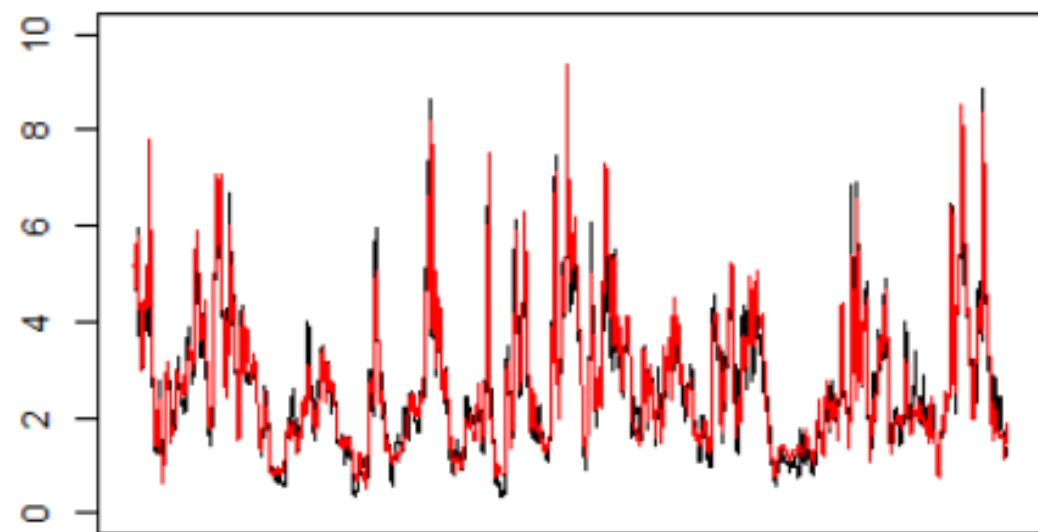
□ In this study, we suppose that  $\hat{\beta}^G$  is smooth and has the same EOFs as  $X^G$  and we choose  $\Delta = (X^G{}^T X^G)^{-1} \alpha$  and  $\lambda$  and  $\alpha$  are selected using cross validation

- The period from 1994 to 2012 is used to estimate the parameters  $\beta^L$  and  $\beta^G$
- 2013 to 2016 is used to select the tuning parameters
- 2016 to 2019 is used as a validation period

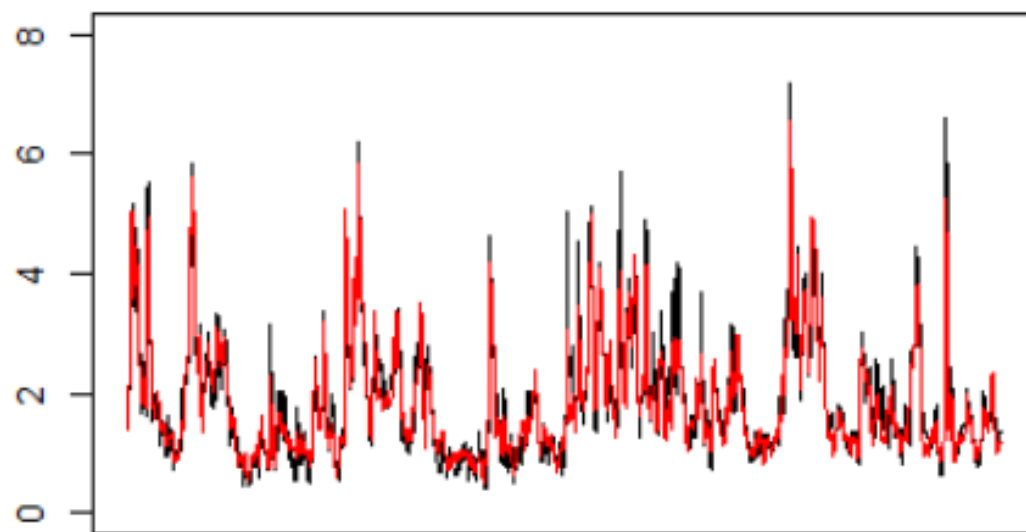
# RESULTS



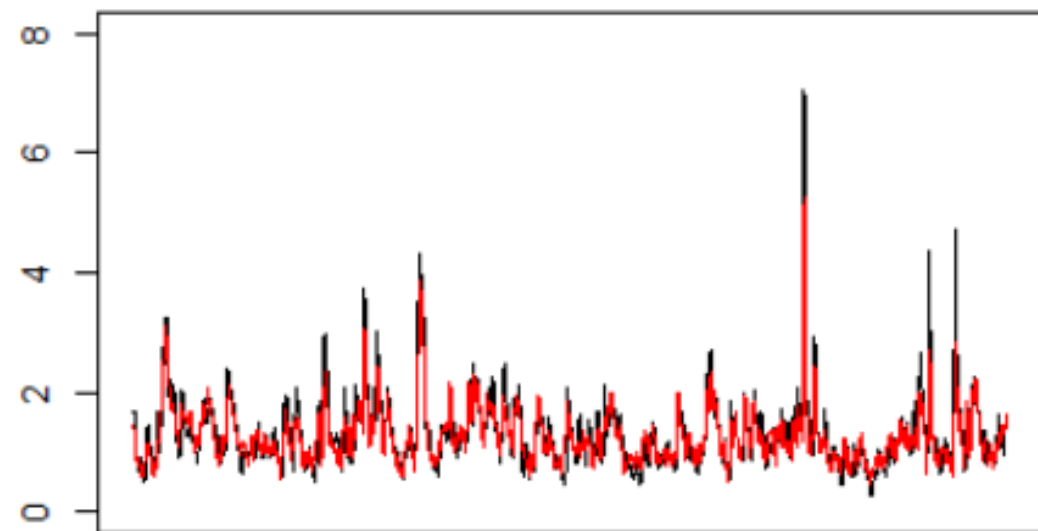
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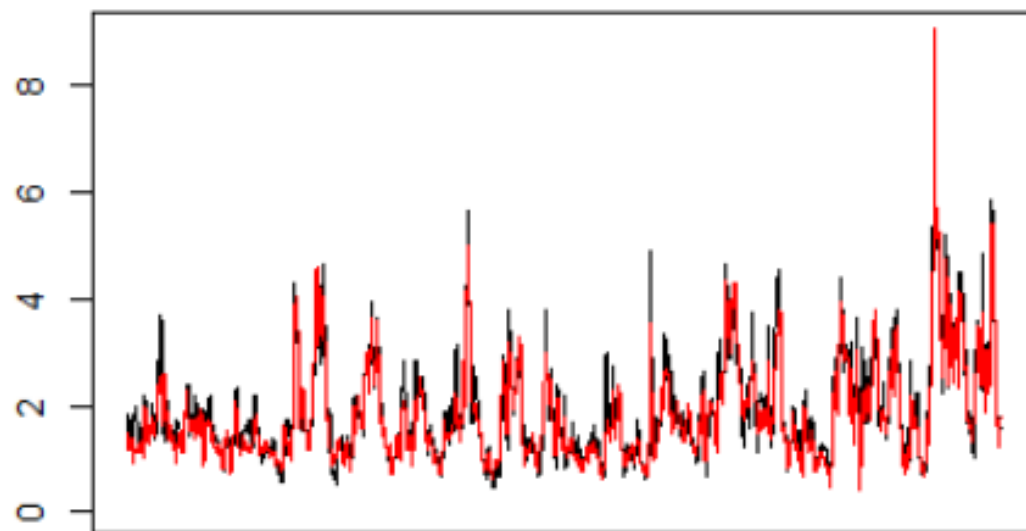
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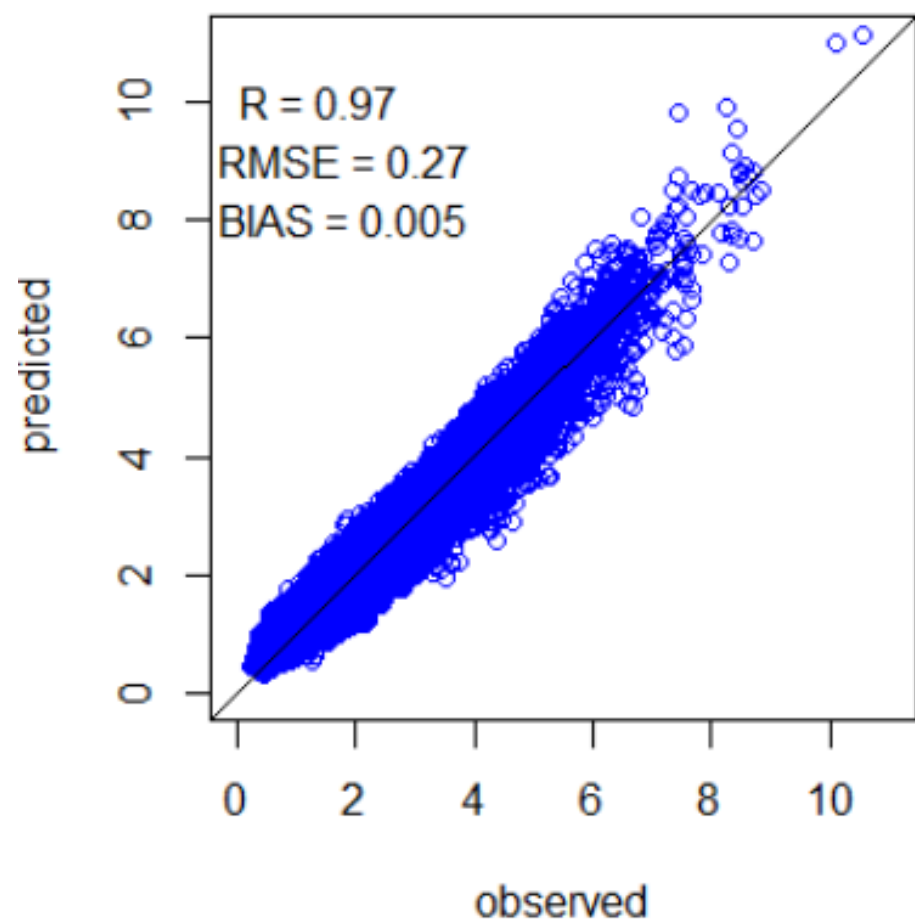
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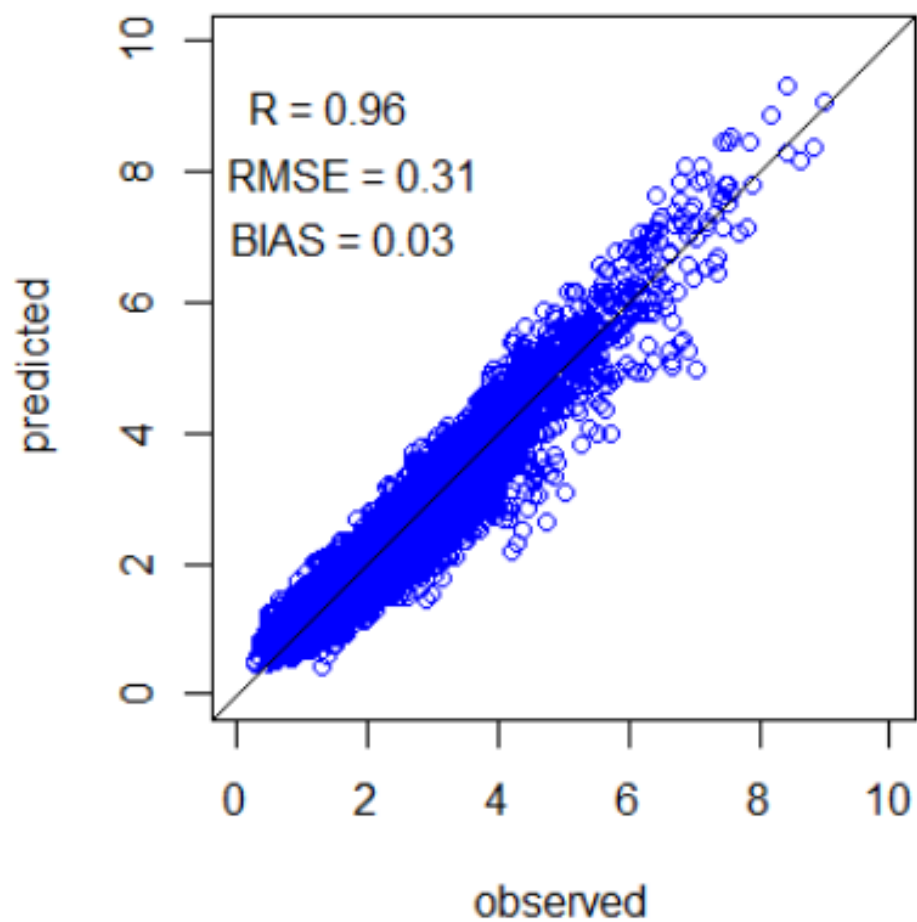
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**Calibration period**



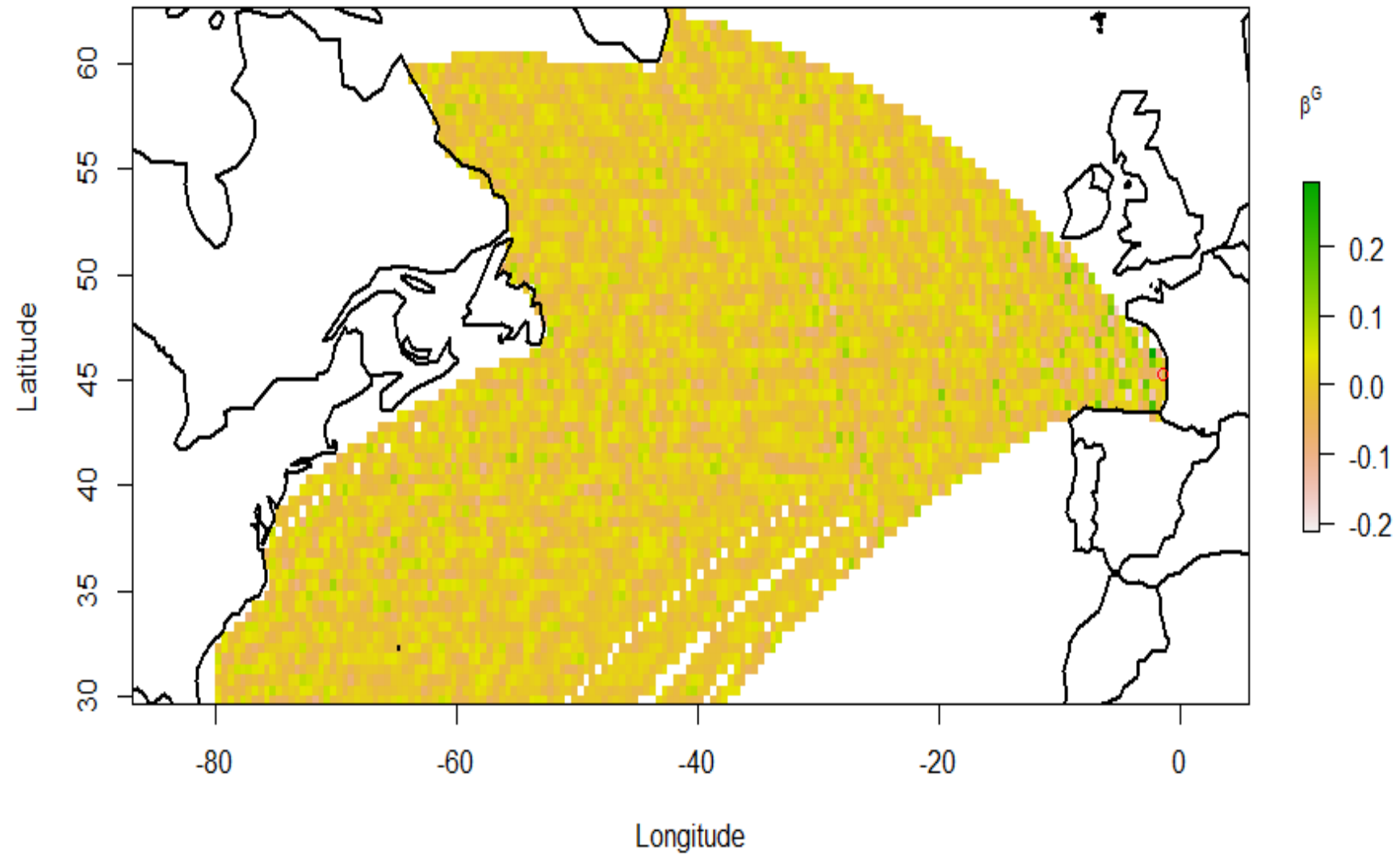
**Validation period**



# COMPARAISON WITH OTHER METHODS

	R	RMSE	BIAS
Ridge	0.96	0.317	0.03
LASSO	0.96	0.318	0.03
Extended Ridge	0.961	0.313	0.03

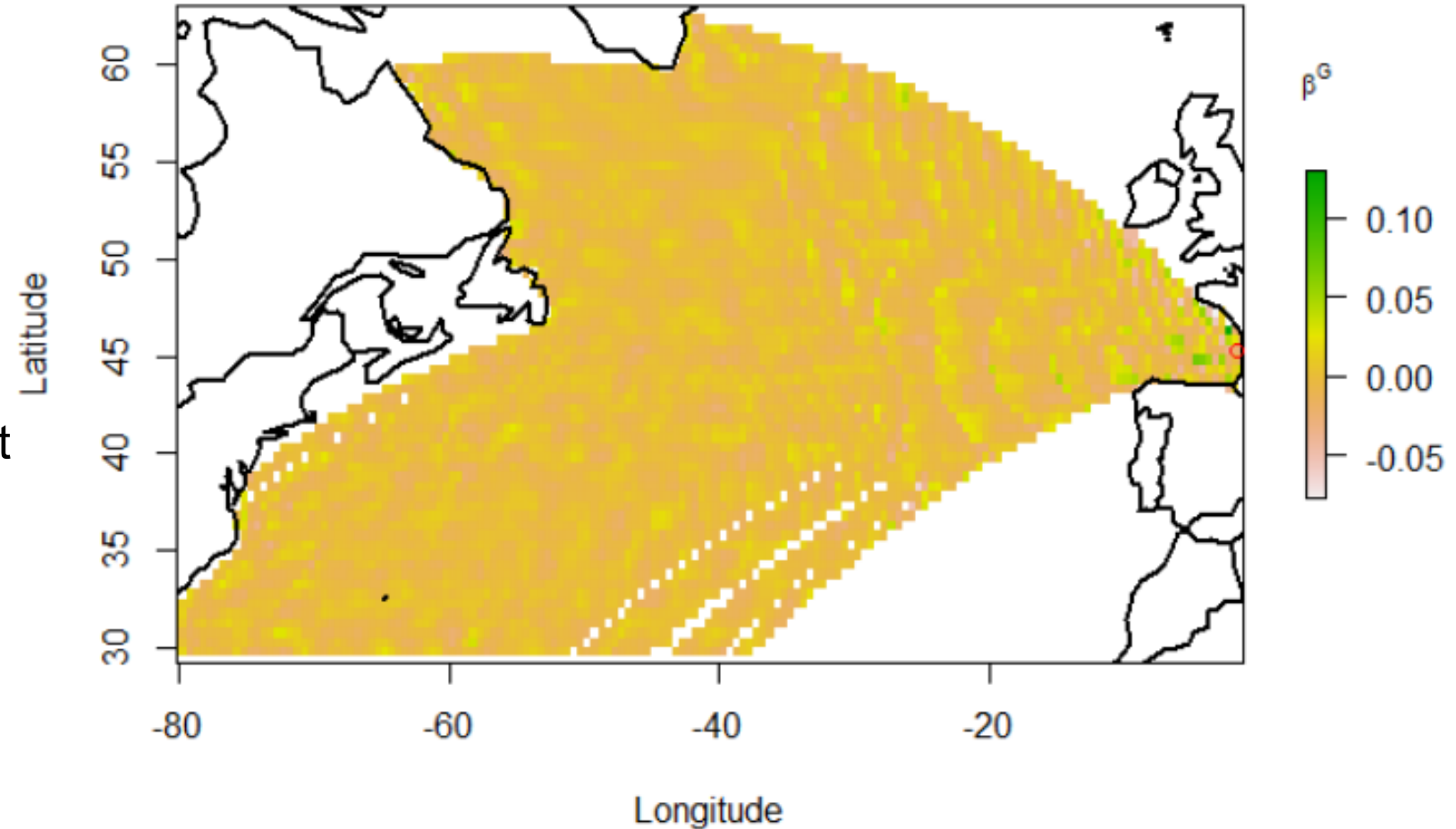
# THE ESTIMATED $\beta$ / $\sigma$ USING LASSO





# THE ESTIMATED $\beta \uparrow G$ USING THE EXTENDED RIDGE

- ❑ Indeed, the  $\lambda$  (the smoothness parameter) choosed by cross validation does not give the desired smoothness
- ❑ By using larger values of  $\lambda$ , the resulted coefficients are smooth however; the prediction accuracy get worse
- ❑ Trade off between model interpretability and model prediction accuracy?



# CONCLUSION

- ❑ A statistical downscaling model that links the large-scale wind and the local-scale wave parameter ( $H_s$ ) was proposed
- ❑ Predictors definition is a crucial step in the statistical downscaling framework
- ❑ The validation analysis proves the model's skill in predicting wave climate
- ❑ Working on the trade off between prediction accuracy and interpretability

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**THANK YOU FOR YOUR ATTENTION**